# OPTICAL INVESTIGATION OF THE STRESS CONCENTRATION IN A SHEET WITH A CIRCULAR OPENING SUBJECTED TO BIAXIAL TENSION

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Below are presented some results of an optical investigation of the stress concentration at the "danger points" A of a plate with a circular opening stressed in tension (Fig. 1) under conditions of steady creep. A relation is found linking the stress concentration factor at the "danger points" with the mechanical properties of the medium and the nature of the stress state "at infinity." The results are also applicable by analogy to a plastic medium with strain hardening.

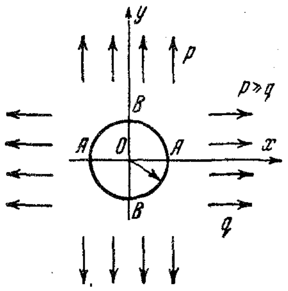


Fig. 1

A theoretical solution of the problem (Fig. 1) for the axisymmetric case, when the stresses at infinity are equal (p = q), was obtained by A. G. Kostyuk [1] for a medium with a power law of strain hardening

$$\varepsilon_i = Bs_i^m \qquad (s_i = {}^3/_2 \sqrt{s_{\alpha\beta}s_{\alpha\beta}}). \tag{1}$$

Here $\varepsilon_i$ is the shear strain rate, $s_i$ is the shear stress intensity, $s_{\alpha\beta}$ are the components of the stress deviator, and B and m are material constants. An approximate solution of the problem for p = q was obtained by V. I. Rozenblyum [2] using the Tresca-Saint Venant potential. Budiansky and Mangasarian [3] give the solution for a Ramberg-Osgood medium.

Attempts to solve the problem for uniaxial tension (q = 0) were made in [4-6].

An experimental solution of the problem, based on a scheme different from that assumed here, was presented in [7-10].

In [11] the problem was solved by the "photocreep" method using models made of two kinds of transparent technical celluloid of different "age." Celluloid 1 had an age of 1.5 years, and celluloid 2 an age of 6 months. From earlier experiments on celluloid in biaxial tension [12] it follows that in the state of steady (or more correctly, quasi-steady [13, 11]) creep the relation

$$\varepsilon_i = \varphi(t) s_i \exp(bs_i) \tag{2}$$

Here t is time, and b a material constant. For small stress intervals the simpler relation (1) is applicable, in which case B is a function of t.

We carried out uniaxial tests on specimens of the model material at constant load and at the same temperature (20°C) as that at which the models were investigated.

The tests showed that for celluloid 1 the constant b = 0.011; for $s_i$ in the interval from 85 to 210 kg/cm² it is possible to take m = 2.2.

For celluloid 2 the constant b = 0.016; for $s_i$ in the interval from 100 to 180 kg/cm² it is possible to take m = 3.

The models had the shape of the crosses [14] used in investigating the properties of celluloid in biaxial tension [12] and in the experimental solution of the problem (Fig. 1) for the case p = q, m = 2.5 [11].

In the center of the crosses, which measured 60 × 60 mm, there was a hole 7 mm in diameter; the thickness of the models was 4 mm.

The models were extended in a test machine that made it possible to apply constant independent stresses p and q to the arms of the cross. Thanks to the longitudinal notches in the arms of the cross, in the absence of a hole an almost uniform, statically determinate stress state is created in the center of the cross.

In the experiments we took the following values for p and q:

| | | | | |
|---|---|---|---|---|
| p, kg/cm² = | 100 | 114 | 114 | 100 |
| q, kg/cm² = | 0 | 38 | 76 | 100 |
| $\alpha = q/p =$ | 0 | $^1/_3$ | $^2/_3$ | 1 |

In all the experiments the shear stresses "at infinity"

$$s_i^\infty = \sqrt{p^2 - pq + q^2}$$

were equal to 100 kg/cm$^2$. The optical path difference $\delta$ was measured by the Senarmon method [11] in plane-polarized light of wavelength 546 m$\mu$. The results obtained for the two points A were averaged. The conversion from $\delta$ to the principal stress $\sigma_1 = \sigma_*$ at the point A was realized by means of isochronous curves in the coordinates $\delta$, $\sigma_1$. These curves were constructed from the results of measurements of $\delta$ in the above-mentioned uniaxial tests on specimens of the model materials. Previous investigations showed that this method of conversion is sufficiently reliable [11]; the error is determined by the technical characteristics of the experiment.

The stress concentration factors k at A were computed from the formula

$$k = \sigma_* / p.$$

Polarization-optical measurements showed that for celluloid 1 stress redistribution is complete about 25 hours after loading the model. The corresponding figure for celluloid 2 is 5 hours. Thereafter the models are in a state of steady creep. The corresponding values of k are:

| | | | | |
|---|---|---|---|---|
| $\alpha = 0$ | $^1/_3$ | $^2/_3$ | 1 | |
| $k = 2.00$ | 1.82 | 1.67 | 1.50 | celluloid 1 |
| $k = 1.72$ | 1.57 | 1.43 | 1.30 | celluloid 2. |

The values of k are shown by circles in Fig. 2; the dots indicate the results of [1] for $\alpha = 1$, $m = 2.2$, and $m = 3$; in this case the line 1 corresponds to the values $s_0 = 0$, $m = 1$; the line 2 – celluloid 1 – to $s_0 = 1.1$, $m = 2.2$; the line 3 – celluloid 2 – to $s_0 = 1.6$, $m = 3$; and the line 4 to $s_0 \to \infty$, $m \to \infty$.

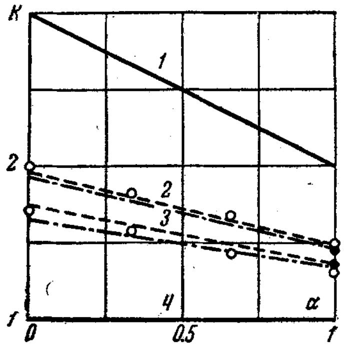

Fig. 2

It may be assumed that for $\alpha = 1$ the theoretical and experimental values of k almost coincide. Good agreement with the data of [1] was previously obtained for $\alpha = 1$, $m = 2.5$ [11].

We introduce the dimensionless parameter $s_0 = bs_i^\infty$.

At m = 1 the stress field coincides with the stress field in the corresponding linear-elastic problem [13] and we have [15] the relation $k = 3 - \alpha$ shown in Fig. 2, line 1.

An analogous situation prevails at $s_0 = 0$, since then $b = 0$; Eq. (2) becomes linear with respect to the relation between $\varepsilon_i$ and $s_i$.

At $m > 1$, $s_0 > 0$ the factor k again has a linear relation with $\alpha$ (Fig. 2).

As $m \to \infty$, a "limiting creep state" develops. If regions of rigid displacement are impossible (e.g., at $\alpha = 1$), the stress field in the "limiting creep state" coincides, for any value of the external loads, with the limiting ideally plastic state [13]. Solution [16] of the ideally plastic problem for $\alpha = 1$ gives $k = 1$. We are unacquainted with possible solutions of analogous problems for $\alpha < 1$. However, there is reason to suppose that $k = 1$ for all possible $\alpha$ (horizontal axis in Fig. 2).

An analogous picture is also observed for $s_0 \to \infty$.

For the stress concentration factor at points A we get the simple approximation formulas

$$k = 1 + \frac{2-\alpha}{1+s_0}, \qquad k = 1 + \frac{2-\alpha}{m}. \tag{3}$$

The first of these is used in conjunction with relation (1), the second in conjunction with relation (2).

At $s_0 = 0$ and $s_0 \to \infty$ and analogously at $m = 1$ and $m \to \infty$ formulas (3) become exact. Their use at finite values of $s_0 > 0$ and $m > 1$ leads to good results (Fig. 2, broken and dash-dot lines, respectively).

The second of Eqs. (3) is a generalization of the relation $k = 1 + 1/m$ for $\alpha = 1$, proposed by A. G. Kostyuk, which is in good agreement with the exact solution:

| | | | | | |
|---|---|---|---|---|---|
| $m = 1$ | 1.67 | 3 | 5 | $\infty$ | |
| $k = 2$ | 1.62 | 1.35 | 1.21 | 1 | [1] |
| $k = 1 + 1/m = 2$ | 1.60 | 1.33 | 1.20 | 1. | |

At $\alpha \geq 2/3$ the shear stress intensity in the models lies within the range in which the assumed values of m hold true. For $\alpha$ in the range $0 < \alpha < 2/3$ at points B (Fig. 1) there is a local zone with a low level of $s_i$. Here the values of m differ from those assumed. The good agreement between the results of calculations based on the second of Eqs. (3) and the experimental data shows that this has little effect on the value of k at A and, consequently, that using (1) at $\alpha < 2/3$ leads to satisfactory results.

Further, at $\alpha < 1/3$, under conditions of elasticity and creep, there is a local zone of compressive tangential stresses at points B. It is assumed that the properties of the medium in tension and compression are the same or that an existing difference does not affect the value of k at A.

There is reason to suppose that Eqs. (3) also hold true for $\alpha < 0$, in particular, in the case of pure shear ($q = -p$, $\alpha = -1$).

Equations (3) may be used for the approximate calculation of the stress concentration factors at the "danger points" in turbine disks with small openings. The role of the stresses p and q is played by the radial and tangential stresses at points of the disk where the openings are located, but without allowance for the latter.

The maximum tensile stresses do not act at the edge of the opening for all values of $s_0$ and m. Starting from a certain value of $s_0$ and m the "danger points" are displaced inside the region in the direction 0x (Fig. 1). However, the maximum tensile stress at these points evidently differs only slightly from that at A. Thus, in the case of the axisymmetric problem for the "limiting creep state" the "danger points" are located on the radius $r \approx 2.07a$ ($a$ is the radius of the opening), and the stress at these points is 15% greater than that at the edge of the opening [16]. In our experiments the maximum tensile stress was observed at the edge of the opening.

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